EP155 February 6, 2006 Midterm #1

Name: SOLUTION Student No. ______________

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Date: February 6, 2006
Time: 1 hour
Restrictions: Calculators and one 8.5 by 11 sheet of paper only.
              The sheet of paper can be written on both sides.
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Put a box around all your answers!

Show the units!

CONSTANTS:
$k = 8.99 \times 10^9\ \mathrm{N\,m^2/C^2}$
$\epsilon_o = 8.85 \times 10^{-12}\ \mathrm{C^2/(N\,m^2)}$
PREFIXES:
μ is 10^{-6}
m is 10^{-3}
k is 10^3

QUES.	MARKS
Q1 (6)	
Q2 (8)	
Q3 (6)	
Q4 (6)	
Total (26)	

(2) 1. (a) An object is moved from point (x,y)=(7 m, 49 m) to point (x,y) = (19 m, 9 m). The object is moved with a constant force vector

$$\vec{F} = (10\hat{\imath} - 7\hat{\jmath})\ \text{N}.$$

How much work is done on the object by the force?

$W = \vec{F} \cdot \vec{d}$; $\vec{d}$ = final position − initial position

$\vec{d} = (19m, 9m) - (7m, 49m) = (12m, -40m)$

$W = (10N, -7N) \cdot (12m, -40m)$

$W = (10N)(12m) + (-7N)(-40m)$

$W = 120J + 280J$

$\boxed{W = 400J}$

(4) (b) An object is moved in a straight line from point A, which is located at (x,y) = (0 m, 0 m) to point B, which is located at (x,y)=(10 m, 0 m). It is then moved in another straight line from point B to point D, which is located at (x,y)=(10 m, 6 m). The object is moved with a constant force vector $\vec{F}$. The force does 10 J of work to move the object from point A to point B and 12 J of work to move the object from point B to point D.
What is the force vector $\vec{F}$?

Let $\vec{F} = F_x\hat{\imath} + F_y\hat{\jmath}$

The distance vector for moving from A to B is

$\vec{d}_1 = (10m, 0m) - (0m, 0m) = (10m, 0m)$

The work done in moving the object from A to B is given by:

$W = \vec{F} \cdot \vec{d}_1 = (F_x, F_y) \cdot (10m, 0m)$

$10J = (F_x)(10m) + (F_y)(0m)$

$\therefore \boxed{F_x = \frac{10J}{10m} = 1N}$

The distance vector for moving from B to D is

$\vec{d}_2 = (10m, 6m) - (10m, 0m) = (0m, 6m)$

The work done in moving the object from B to D is

$W = \vec{F} \cdot \vec{d}_2 = (F_x, F_y) \cdot (0m, 6m)$

$12J = (F_x)(0) + (F_y)(6m)$

$F_y = \frac{12J}{6m} = 2N$

$\boxed{\vec{F} = (1N, 2N) = 1N\hat{\imath} + 2N\hat{\jmath}}$

2. The field lines for an electric field are shown in Figure 1. There are several points shown on the field. It is known that the electric field strengths at points A, B and D are 10 N/C, 12 N/C and 14 N/C respectively.

(2) (a) Draw an equipotential line (i.e., an energy contour line) through point A.

(2) (b) The distance between points A and B is 7 m (the distance is measured along the field line that links them) and the distance between points B and D is 5 m (the distance is measured along the field line that links them).

Approximately what is the electic potential at point A with respect to point D (i.e., Approximately what is V_{AD})?

$V_{AD} = \frac{W}{Q_t}$ ← work required to move Q_t from point D to point A.

First determine the sign of V_{AD}. If Q_t is positive the field will exert a force to the right so the mover must do positive work $\therefore \boxed{V_{AD}\ +}$

$\frac{W}{Q_t} = \frac{W_1}{Q_t} + \frac{W_2}{Q_t}$; W_1 is work required to move Q_t from D to B. W_2 is work required to move Q_t from B to A.

$\frac{W_1}{Q_t} = \left(\frac{F_{ave}}{Q_t}\right)(d) = E_{ave}\, d = \left(\frac{12+14}{2}\ \frac{N}{C}\right) 5\ m = 65\ \frac{J}{C} = 65V.$

$\frac{W_2}{Q_t} = \left(\frac{F_{ave}}{Q_t}\right)(d) = E_{ave}\, d = \left(\frac{10+12}{2}\right)\frac{N}{C}\ 7m = 77\ \frac{J}{C} = 77V.$

$\therefore \boxed{V_{AD} = \frac{W_1}{Q_t} + \frac{W_2}{Q_t} = 65V + 77V = 142V}$

(2) (c) How much work is required to move +9 C of charge from point A to point D.

$V_{DA} = \frac{W}{Q_t}$ ← work required to move Q_t from A to D.

Positive charge is moved in direction of field line therefore the work will be negative

$W = V_{DA}\, Q_t = -V_{AD}\, Q_t = (-142V)(9\ C) = -1278J$

$\boxed{W = -1278J}$

(2) (d) Approximately what is V_{AG}?

To find V_{AG} draw a equipotential line through point G to determine where it intersects the field line that has point A, B, and C. Then calculate (roughly) the work required to move Q_t from the intersection to point A.

$V_{AG} = \frac{W}{Q_t}$ ← work to move Q_t from intersect to point A.

On drawing the equipotential line through point G, it is found that it intersects the top field line at point B.

$\therefore V_{AG} = V_{AB} = \frac{W}{Q_t} = \left(\frac{F_{ave}}{Q_t}\right)$ (distance from B to A).

$\boxed{V_{AG} = \left(\frac{10+12}{2}\right)\frac{N}{C}\ (7m) = 77V}$

check the sign. The field force (on positive Q_t) is to the right and the charge is moved to the left $\therefore$ work done is positive.

$\therefore \boxed{V_{AG}\ +}$ This checks ✓

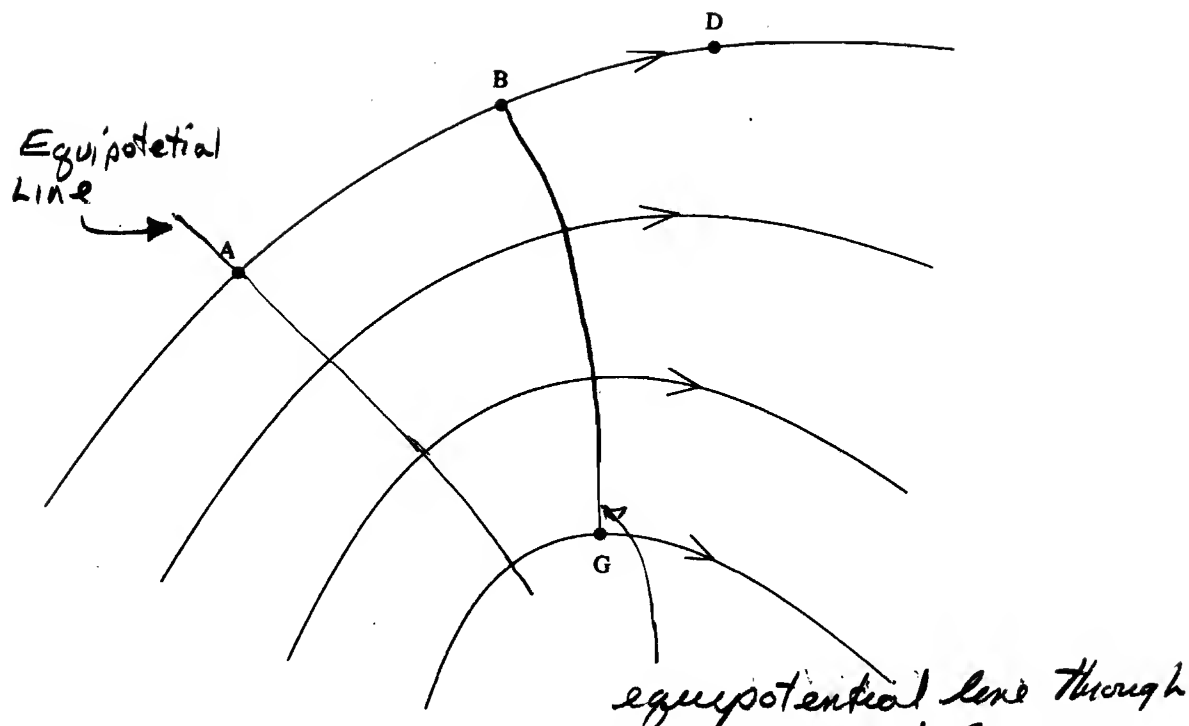


Figure 1: The electric field lines for an electric field.

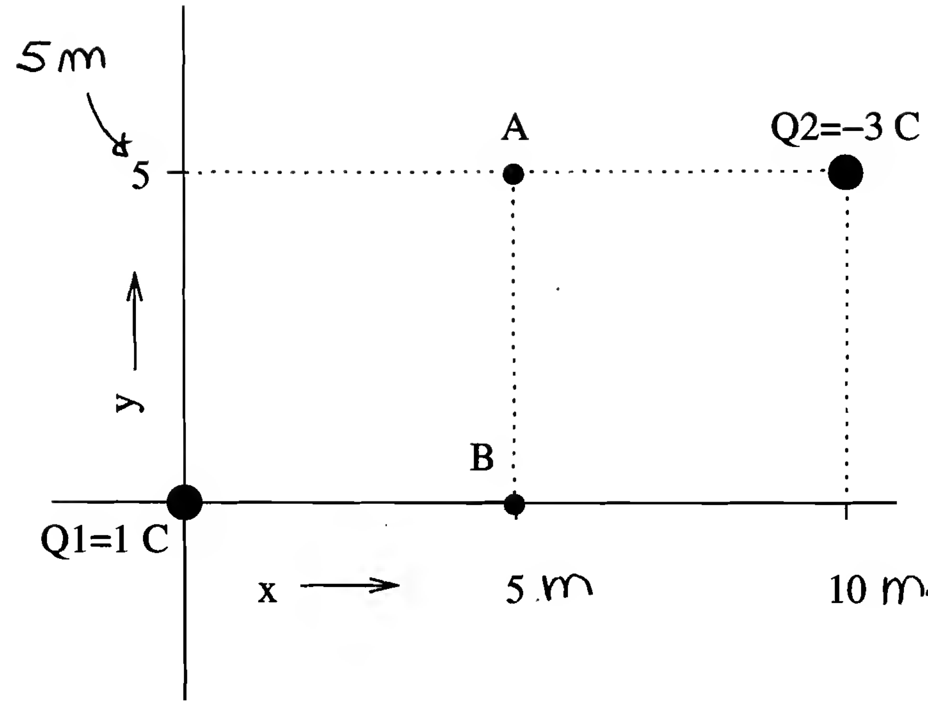


Figure 2: Position of point charges Q1 and Q2 as well as points A and B.

3. Two charge particles are positioned as shown in Figure 2. One particle is charged with Q1 = +1C and the other is charged with Q2 = -3C.

(2) (a) If a test charge of Q_t = +1 C is placed at point A, what force would it experience due the field set up by point charges Q1 and Q2? The force will be a vector.

Let the force on Q_t be denoted $\vec{F} = F_x \hat{i} + F_y \hat{i}$.
Let $\vec{F_1} = F_{1x}\hat{i} + F_{1y}\hat{j}$ be the component of $\vec{F}$ due to charge Q_2 and let $\vec{F_2} = F_{2x}\hat{i} + F_{2y}\hat{j}$ be the component of force due to Q_2. We can get $|\vec{F_1}|$ and $|\vec{F_2}|$ using Coulombs law.

$|\vec{F_1}| = \frac{k Q_1 Q_t}{r_1^2}$ where r_1 is distance between Q_1 & Q_t, $r_1 = \sqrt{5^2+5^2}$
$r_1 = \sqrt{2}\,5m$.

$$|\vec{F_1}| = \frac{\left(8.99\times10^9 \frac{Nm^2}{C}\right)(1C)(1C)}{(\sqrt{2}\,5m)^2} = 0.180\times10^9 N$$

$$|\vec{F_2}| = \frac{k Q_2 Q_t}{r_2^2} = \frac{\left(8.99\times10^9 \frac{Nm^2}{C}\right)(3C)(1)}{(5m)^2} = 1.079\times10^9 N$$

Since Q_1 & Q_t are both positive, the force due to Q_1 is repulsive. From the geometry $F_{1x} = F_{1y}$ and both are positive.

$\therefore \vec{F_1} = F_{1x}\hat{i} + F_{1x}\hat{j}$ so that $|\vec{F_1}| = \sqrt{F_{1x}^2 + F_{1y}^2} = \sqrt{2}\,F_{1x}$.

$\therefore F_{1x} = F_{1y} = \frac{|\vec{F_1}|}{\sqrt{2}} = \frac{0.180\times10^9}{\sqrt{2}} = 0.127\times10^9 N$

Since Q_2 & Q_t are of opposite signs the force is one of attraction $\therefore$

$\vec{F_2} = |F_2|\hat{i} = 1.079\times10^9 N\,\hat{i}$

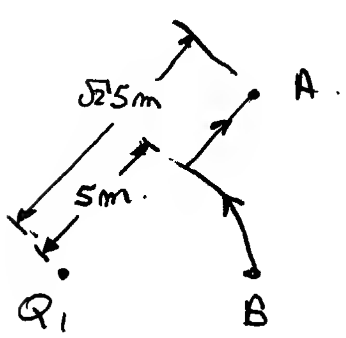
$$\boxed{\vec{F} = F_1 + F_2 = 1.206\times10^9 N\,\hat{i} + 0.127\times10^9 N\,\hat{j}}$$

(4) (b) What is V_{AB}?

$V_{AB} = \frac{W}{Q_t}$ ← work done in moving Q_t from point B to point A.

It is easiest to find W in two steps. Find the work done to overcome the force due to Q_1 and the find the work done to overcome the force due to Q_2 and finally, sum the results to get W.

First find to work done to overcome the force due to Q_1.
Need to choose a sensible path, which is one that either follows an equipotential line or a field line.
The path: Start at point B, move counterclockwise around a semicircle with center at Q_1 until the field line that goes through point A is reached. Then follow the field line to point B.



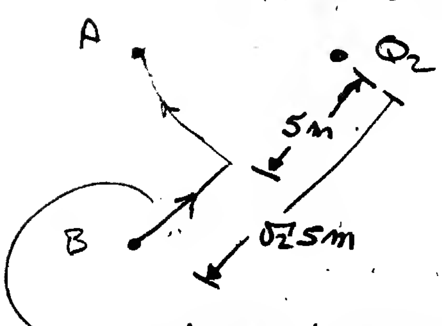
No work is done while moving on the semicircle as the path is perpendicular to the field lines.
The work done to moving Q_t (if Q_t is positive) along the field line is negative as the field is doing the work.
The magnitude of the work done is

$$|W_1| = \left| \int_{r=5m}^{r=\sqrt{2}5m} \frac{K Q_1 Q_t}{r^2}\, dr \right| = K|Q_1||Q_t| \left| \frac{1}{r_i} - \frac{1}{r_f} \right|$$

$$|W_1| = \left(8.99 \times 10^9 \frac{Nm^2}{C^2}\right)(1C)\, Q_t \left| \frac{1}{5m} - \frac{1}{\sqrt{2}5m} \right| = 0.527 \times 10^9 \frac{Nm}{C} |Q_t|.$$

$$W_1 = -0.527 \times 10^9\, V Q_t.$$

For work required to overcome force due to Q_2 use path.



$$|W_2| = \left| \int_{r=\sqrt{2}5m}^{5m} \frac{K Q_2 Q_t}{r^2}\, dr \right| = K|Q_2||Q_t| \left| \frac{1}{r_i} - \frac{1}{r_f} \right|.$$

$$|W_2| = 8.99 \times 10^9 \frac{Nm^2}{C^2}(3C)(Q_t) \left| \frac{1}{\sqrt{2}5m} - \frac{1}{5m} \right|$$

$$|W_2| = 1.580 \times 10^9\, V\, Q_t.$$

work done is negative if Q_t is positive since Q_2 is neg.

$$W_2 = -1.58 \times 10^9\, V Q_t.$$

$$V_{AB} = \frac{W_1 + W_2}{Q_t} = \frac{-0.527 \times 10^9 V \cancel{Q_t} - 1.58 \times 10^9 V \cancel{Q_t}}{\cancel{Q_t}} = -2.107 \times 10^9\, V.$$

$$\boxed{V_{AB} = -2.107 \times 10^9\, V}$$

4. A parallel plate capacitor has the following attributes.

   The area of each plate is 0.15 m^2.

   The distance between the plates is 10^{-5} m.

   The material between the plates is polystyrene (Polystyrene has a dielectric constant of 2.6.).

   The voltage across the plates is 10 V.

(2) (a) What is the capacitance of the capacitor?

$$C = \frac{K \epsilon_0 A}{d} = \frac{(2.6)(8.85\times10^{-12}\frac{C^2}{m^2 N})(0.15\ m^2)}{10^{-5} m}$$

$$C = 0.345\times10^{-6}\ \frac{C^2}{Nm}$$

$$\boxed{C = 0.345\ \mu F}$$

(2) (b) What is the amount of charge (excess charge) on each plate of the capacitor?

$$|Q| = CV = \left(0.345\times10^{-6}\ \frac{C}{V}\right)(10\ V)$$

$$\boxed{|Q| = 3.45\ \mu C}$$

(2) (c) What is the electric field strength between the plates of the capacitor?

field strength, which is constant between the plates

distance between plate

$V = E\,d$

voltage across the plates

$$\boxed{E = \frac{V}{d} = \frac{10\ V}{10^{-5}\ m} = 10^6\ \frac{V}{m} = 10^6\ \frac{N}{C}}$$

Alternate method.

$|Q_{eff}|$

voltage across plate

$$E = \frac{\sigma_{eff}}{\epsilon_0} = \frac{|Q_{eff}|}{\epsilon_0 A} = \frac{\left(\frac{\epsilon_0 A}{d}\right)V}{\epsilon_0 A} = \frac{V}{d}$$

which results in same formula.

$$E = \frac{V}{d}$$

Common Mistakes

Question 1 a)

The most common mistake was taking the dot product incorrectly and getting a vector result.
Work is a scaler and a dot product results in a scaler.

$$W = \vec{F} \cdot \vec{d} = (10, -7) \cdot (12, -40)$$
$$= (10)(12) + (-7)(-40)$$
$$= 120 + 280$$
$$\boxed{W = 400}$$

NOTE:

$$(10, -7) \cdot (12, -4) \neq 120\hat{i} + 280\hat{j}$$

Question 2b

Two different mistakes were often made in computing the average force.

The first was the calculation of average force on charge Q_t when it is moved from point B to point A.

$$W \cong F_{Ave}\, d \; ; \qquad F_{Ave} = \frac{F_B + F_A}{2}$$

$$F_{Ave} \neq \frac{F_B - F_A}{2}$$

The second commonly made mistake was using a less accurate average force in computing the work in moving charge Q_t from point D to point A

$$W = \underbrace{W_1}_{\text{work to move from D to B}} + \underbrace{W_2}_{\text{work to move from B to A}}$$

$$W \cong \left(\frac{F_D + F_B}{2}\right) 5m + \left(\frac{F_B + F_A}{2}\right) 7m = \frac{5F_D + 12F_B + 7F_A}{2}$$

Using average force $\frac{F_D + F_A}{2}$ and distance 12 m yield a less accurate result $W \cong 6F_D + 6F_A$

Question 3b).

The most common mistake was to find the average force incorrectly and then use this average force to find the work done. Note that if

$$F_S(r) = \frac{KQ_1 Q_t}{r^2},$$ then the average force applied from $r = r_i$ to $r = r_f$ is not $$\frac{F_S(r_f) + F_S(r_i)}{2}.$$ The average force is in fact.

$$F_{S\,Ave} = \left(\frac{1}{r_f - r_i}\right) \int_{r_i}^{r_f} \frac{KQ_1 Q_t}{r^2}\,dr$$

$$F_{S\,Ave} = \left(\frac{KQ_1 Q_t}{r_f - r_i}\right)\left[\frac{1}{r_i} - \frac{1}{r_f}\right]$$

$$W = F_{S\,Ave}(r)\,\overset{r_f - r_i}{d}$$

$$W = KQ_1 Q_t \left[\frac{1}{r_i} - \frac{1}{r_f}\right]$$

NOTE: You were given this formula for work and did not need to find $F_{S\,AVE}$

Question 4 c)

Several students used the formula for the energy stored in a capacitor to compute the electric field strength.

Perhaps they used the letter E to represent both electric field strength and energy so had two formula's on their sheet that contained E and picked the wrong one in the heat of the battle. (I am just guessing that this may be the case.)